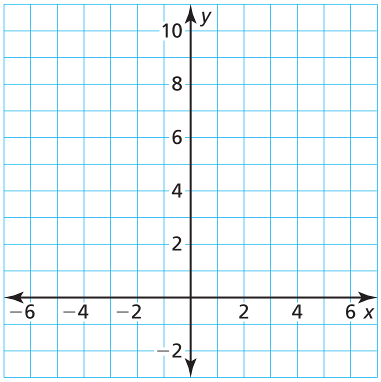
8.2 Graphing $f(x) = ax^2 + c$

Essential Question How does the value of c affect the graph of $f(x) = ax^2 + c$?

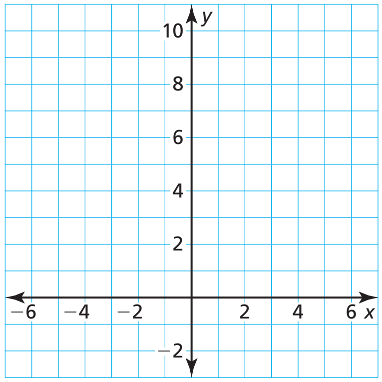
EXPLORATION 1 Graphing $y = ax^2 + c$

Work with a partner. Sketch the graphs of the functions in the same coordinate plane. What do you notice?

a. $f(x) = x^2$ and $g(x) = x^2 + 2$



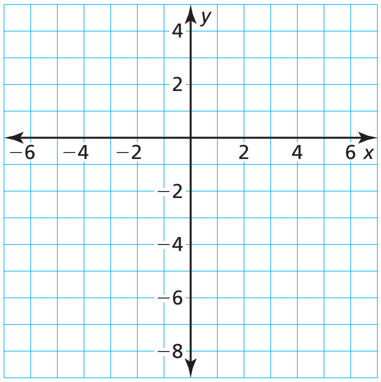
b. $f(x) = 2x^2$ and $g(x) = 2x^2 - 2$



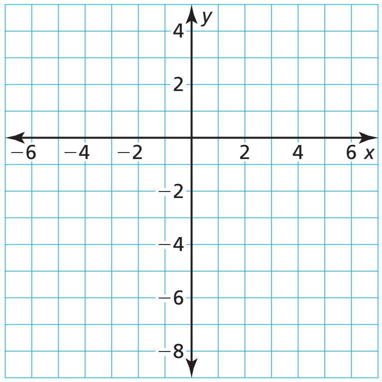
EXPLORATION 2 Finding x-Intercepts of Graphs

Work with a partner. Graph each function. Find the x-intercepts of the graph. Explain how you found the x-intercepts.

a. $y = x^2 - 7$



b. $y = -x^2 + 1$



USING TOOLS STRATEGICALLY

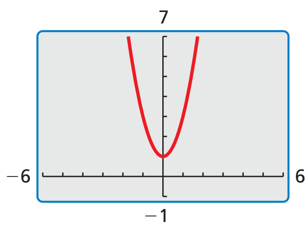
To be proficient in math, you need to consider the available tools, such as a graphing calculator, when solving a mathematical problem.

Communicate Your Answer

3. How does the value of c affect the graph of $f(x) = ax^2 + c$?

4. Use a graphing calculator to verify your answers to Question 3.

5. The figure shows the graph of a quadratic function of the form $y = ax^2 + c$. Describe possible values of a and c. Explain your reasoning.



8.2 Lesson

Core Vocabulary

zero of a function, *p. 428*

Previous
translation
vertex of a parabola
axis of symmetry
vertical stretch
vertical shrink

What You Will Learn

- Graph quadratic functions of the form $f(x) = ax^2 + c$.
- Solve real-life problems involving functions of the form $f(x) = ax^2 + c$.

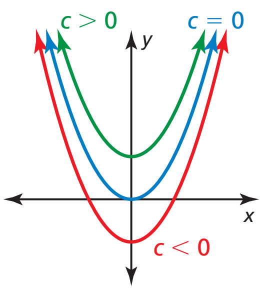
Graphing $f(x) = ax^2 + c$

Core Concept

Graphing $f(x) = ax^2 + c$

- When $c > 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation c units up of the graph of $f(x) = ax^2$.
- When $c < 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation $|c|$ units down of the graph of $f(x) = ax^2$.

The vertex of the graph of $f(x) = ax^2 + c$ is $(0, c)$, and the axis of symmetry is $x = 0$.



EXAMPLE 1 Graphing $y = x^2 + c$

Graph $g(x) = x^2 - 2$. Compare the graph to the graph of $f(x) = x^2$.

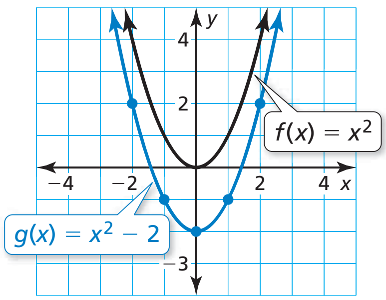
SOLUTION

Step 1 Make a table of values.

x	−2	−1	0	1	2
$g(x)$	2	−1	−2	−1	2

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



REMEMBER

The graph of $y = f(x) + k$ is a vertical translation, and the graph of $y = f(x - h)$ is a horizontal translation of the graph of f.

Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of the graph of g, $(0, -2)$, is below the vertex of the graph of f, $(0, 0)$, because the graph of g is a vertical translation 2 units down of the graph of f.

Monitoring Progress

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Graph the function. Compare the graph to the graph of $f(x) = x^2$.

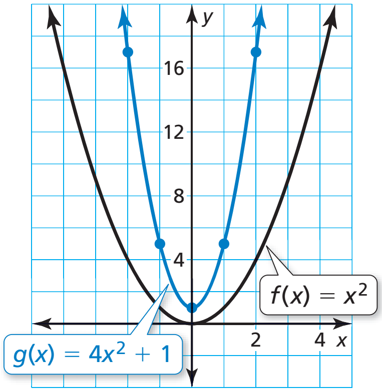
1. $g(x) = x^2 - 5$

2. $h(x) = x^2 + 3$

EXAMPLE 2 Graphing $y = ax^2 + c$

Graph $g(x) = 4x^2 + 1$. Compare the graph to the graph of $f(x) = x^2$.

SOLUTION



Step 1 Make a table of values.

x	-2	-1	0	1	2
$g(x)$	17	5	1	5	17

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

▶ Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of g is narrower, and its vertex, (0, 1), is above the vertex of the graph of f, (0, 0). So, the graph of g is a vertical stretch by a factor of 4 and a vertical translation 1 unit up of the graph of f.

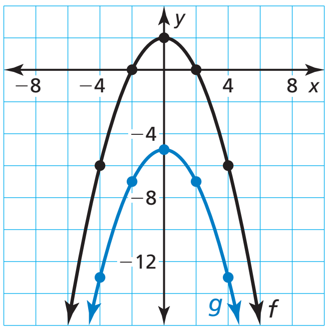
EXAMPLE 3 Translating the Graph of $y = ax^2 + c$

Let $f(x) = -0.5x^2 + 2$ and $g(x) = f(x) - 7$.

a. Describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane.

b. Write an equation that represents g in terms of x.

SOLUTION



a. The function g is of the form $y = f(x) + k$, where $k = -7$. So, the graph of g is a vertical translation 7 units down of the graph of f.

x	-4	-2	0	2	4
$f(x)$	-6	0	2	0	-6
$g(x)$	-13	-7	-5	-7	-13

$-0.5x^2 + 2$

$f(x) - 7$

b. $g(x) = f(x) - 7$ — Write the function g.

$= -0.5x^2 + 2 - 7$ — Substitute for $f(x)$.

$= -0.5x^2 - 5$ — Subtract.

▶ So, the equation $g(x) = -0.5x^2 - 5$ represents g in terms of x.

Monitoring Progress

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Graph the function. Compare the graph to the graph of $f(x) = x^2$.

3. $g(x) = 2x^2 - 5$

4. $h(x) = -\frac{1}{4}x^2 + 4$

5. Let $f(x) = 3x^2 - 1$ and $g(x) = f(x) + 3$.

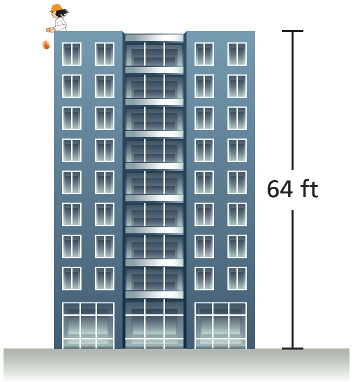
a. Describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane.

b. Write an equation that represents g in terms of x.

Solving Real-Life Problems

A **zero of a function** f is an x-value for which $f(x) = 0$. A zero of a function is an x-intercept of the graph of the function.

EXAMPLE 4 Solving a Real-Life Problem



The function $f(t) = -16t^2 + s_0$ represents the approximate height (in feet) of a falling object t seconds after it is dropped from an initial height s_0 (in feet). An egg is dropped from a height of 64 feet.

a. After how many seconds does the egg hit the ground?

b. Suppose the initial height is adjusted by k feet. How will this affect part (a)?

SOLUTION

1. **Understand the Problem** You know the function that models the height of a falling object and the initial height of an egg. You are asked to find how many seconds it takes the egg to hit the ground when dropped from the initial height. Then you need to describe how a change in the initial height affects how long it takes the egg to hit the ground.

2. **Make a Plan** Use the initial height to write a function that models the height of the egg. Use a table to graph the function. Find the zero(s) of the function to answer the question. Then explain how vertical translations of the graph affect the zero(s) of the function.

3. **Solve the Problem**

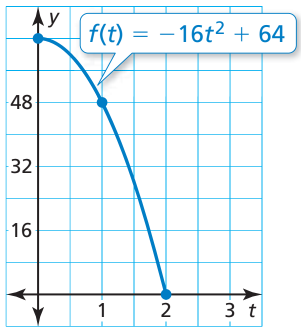
COMMON ERROR

The graph in Step 1 shows the height of the object over time, not the path of the object.

a. The initial height is 64 feet. So, the function $f(t) = -16t^2 + 64$ represents the height of the egg t seconds after it is dropped. The egg hits the ground when $f(t) = 0$.

Step 1 Make a table of values and sketch the graph.

t	0	1	2
$f(t)$	64	48	0



Step 2 Find the positive zero of the function. When $t = 2$, $f(t) = 0$. So, the zero is 2.

▶ The egg hits the ground 2 seconds after it is dropped.

b. When the initial height is adjusted by k feet, the graph of f is translated up k units when $k > 0$ or down $|k|$ units when $k < 0$. So, the x-intercept of the graph of f will move right when $k > 0$ or left when $k < 0$.

▶ When $k > 0$, the egg will take more than 2 seconds to hit the ground. When $k < 0$, the egg will take less than 2 seconds to hit the ground.

4. **Look Back** To check that the egg hits the ground 2 seconds after it is dropped, you can solve $0 = -16t^2 + 64$ by factoring.

Monitoring Progress

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6. Explain why only nonnegative values of t are used in Example 4.

7. **WHAT IF?** The egg is dropped from a height of 100 feet. After how many seconds does the egg hit the ground?

8.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** State the vertex and axis of symmetry of the graph of $y = ax^2 + c$.

2. **WRITING** How does the graph of $y = ax^2 + c$ compare to the graph of $y = ax^2$?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Example 1.)*

3. $g(x) = x^2 + 6$

4. $h(x) = x^2 + 8$

5. $p(x) = x^2 - 3$

6. $q(x) = x^2 - 1$

In Exercises 7–12, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Example 2.)*

7. $g(x) = -x^2 + 3$

8. $h(x) = -x^2 - 7$

9. $s(x) = 2x^2 - 4$

10. $t(x) = -3x^2 + 1$

11. $p(x) = -\frac{1}{3}x^2 - 2$

12. $q(x) = \frac{1}{2}x^2 + 6$

In Exercises 13–16, describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane. Write an equation that represents g in terms of x. *(See Example 3.)*

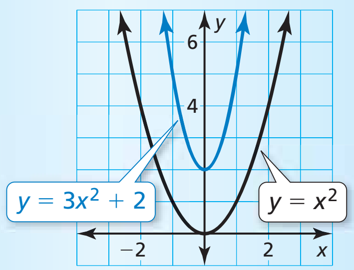
13. $f(x) = 3x^2 + 4$
    $g(x) = f(x) + 2$

14. $f(x) = \frac{1}{2}x^2 + 1$
    $g(x) = f(x) - 4$

15. $f(x) = -\frac{1}{4}x^2 - 6$
    $g(x) = f(x) - 3$

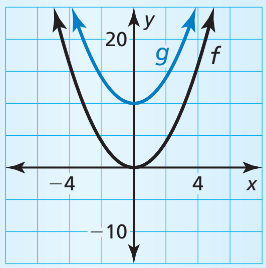
16. $f(x) = 4x^2 - 5$
    $g(x) = f(x) + 7$

17. **ERROR ANALYSIS** Describe and correct the error in comparing the graphs.



The graph of $y = 3x^2 + 2$ is a vertical shrink by a factor of 3 and a translation 2 units up of the graph of $y = x^2$.

18. **ERROR ANALYSIS** Describe and correct the error in graphing and comparing $f(x) = x^2$ and $g(x) = x^2 - 10$.



Both graphs open up and have the same axis of symmetry. However, the vertex of the graph of g, $(0, 10)$, is 10 units above the vertex of the graph of f, $(0, 0)$.

In Exercises 19–26, find the zeros of the function.

19. $y = x^2 - 1$

20. $y = x^2 - 36$

21. $f(x) = -x^2 + 25$

22. $f(x) = -x^2 + 49$

23. $f(x) = 4x^2 - 16$

24. $f(x) = 3x^2 - 27$

25. $f(x) = -12x^2 + 3$

26. $f(x) = -8x^2 + 98$

27. **MODELING WITH MATHEMATICS** A water balloon is dropped from a height of 144 feet. *(See Example 4.)*
    a. After how many seconds does the water balloon hit the ground?
    b. Suppose the initial height is adjusted by k feet. How does this affect part (a)?

28. **MODELING WITH MATHEMATICS** The function $y = -16x^2 + 36$ represents the height y (in feet) of an apple x seconds after falling from a tree. Find and interpret the x- and y-intercepts.

In Exercises 29–32, sketch a parabola with the given characteristics.

29. The parabola opens up, and the vertex is (0, 3).

30. The vertex is (0, 4), and one of the x-intercepts is 2.

31. The related function is increasing when $x < 0$, and the zeros are -1 and 1.

32. The highest point on the parabola is (0, -5).

33. **DRAWING CONCLUSIONS** You and your friend both drop a ball at the same time. The function $h(x) = -16x^2 + 256$ represents the height (in feet) of your ball after x seconds. The function $g(x) = -16x^2 + 300$ represents the height (in feet) of your friend's ball after x seconds.

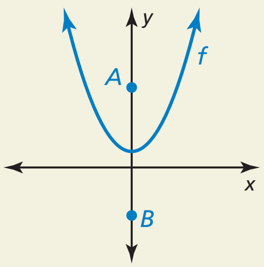
a. Write the function $T(x) = h(x) - g(x)$. What does $T(x)$ represent?

b. When your ball hits the ground, what is the height of your friend's ball? Use a graph to justify your answer.

34. **MAKING AN ARGUMENT** Your friend claims that in the equation $y = ax^2 + c$, the vertex changes when the value of a changes. Is your friend correct? Explain your reasoning.

35. **MATHEMATICAL CONNECTIONS** The area A (in square feet) of a square patio is represented by $A = x^2$, where x is the length of one side of the patio. You add 48 square feet to the patio, resulting in a total area of 192 square feet. What are the dimensions of the original patio? Use a graph to justify your answer.

36. **HOW DO YOU SEE IT?** The graph of $f(x) = ax^2 + c$ is shown. Points A and B are the same distance from the vertex of the graph of f. Which point is closer to the vertex of the graph of f as c increases?



37. **REASONING** Describe two methods you can use to find the zeros of the function $f(t) = -16t^2 + 400$. Check your answer by graphing.

38. **PROBLEM SOLVING** The paths of water from three different garden waterfalls are given below. Each function gives the height h (in feet) and the horizontal distance d (in feet) of the water.

Waterfall 1 $h = -3.1d^2 + 4.8$

Waterfall 2 $h = -3.5d^2 + 1.9$

Waterfall 3 $h = -1.1d^2 + 1.6$

a. Which waterfall drops water from the highest point?

b. Which waterfall follows the narrowest path?

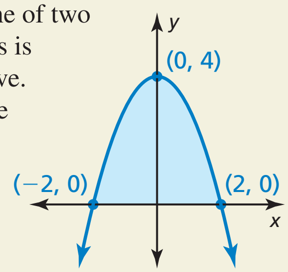
c. Which waterfall sends water the farthest?

39. **WRITING EQUATIONS** Two acorns fall to the ground from an oak tree. One falls 45 feet, while the other falls 32 feet.

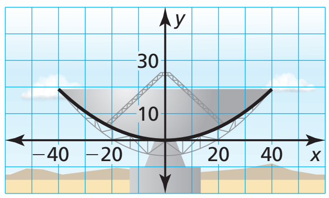
a. For each acorn, write an equation that represents the height h (in feet) as a function of the time t (in seconds).

b. Describe how the graphs of the two equations are related.

40. **THOUGHT PROVOKING** One of two classic problems in calculus is to find the area under a curve. Approximate the area of the region bounded by the parabola and the x-axis. Show your work.



41. **CRITICAL THINKING** A cross section of the parabolic surface of the antenna shown can be modeled by $y = 0.012x^2$, where x and y are measured in feet. The antenna is moved up so that the outer edges of the dish are 25 feet above the x-axis. Where is the vertex of the cross section located? Explain.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when $a = 4$ and $b = -3$. *(Skills Review Handbook)*

42. $\dfrac{a}{4b}$ **43.** $-\dfrac{b}{2a}$ **44.** $\dfrac{a - b}{3a + b}$ **45.** $-\dfrac{b + 2a}{ab}$